Fundamental Numerical Simulation for Disk-shaped Magnetohydrodynamics Accelerator

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Abstract

The purpose of this paper is to investigate the acceleration performance of a disk-shaped magnetohydrodynamics (MHD) accelerator and discuss its MHD acceleration effect. Quasi-1-dimensional (Q1D) numerical simulation employing the MacCormack scheme was developed. For shorter channel lengths, thermal loss can be reduced below 20 % owing to the smaller heat and the friction losses. However, too short channel length, accelerator performance was decreased by the MHD compression due to too large Faraday current density. Generation mechanism of MHD compression in this work was density gradient. Therefore it is necessary to design for “stable acceleration” along the MHD channel.

Keywords

Magnetohydrodynamics; Numerical Simulation; Air-plasma; Disk-shaped

Introduction

Although chemical rocket technology for launching satellites is the most reliable in recent years, it has some issues of high cost and fewer payloads due to low specific impulse. Therefore aerospace researchers have turned their attention to reduce the budget for launching and to increase specific impulse. Magnetohydrodynamics (MHD) accelerator is one of the significant candidates for space propulsion system using scramjet MHD bypass system [1]-[8]. This MHD bypass system is quite simply principle and useful because part of MHD accelerator can generate considerably high acceleration and high speed of plasma due largely to the interaction of the working gas and the applied magnetic field when supplying the electric field (extracted energy from MHD generator part). However, it is found that most of the currently published works are focused on the linear MHD accelerator [1]-[8], which has some disadvantages. For a major example, the length of the channel is quite long. Hence, it is not attractive as it will be carried on-board for space propulsion applications. The aim of this work is to investigate acceleration performance of disk MHD accelerator for MHD channels of various channel length and its MHD acceleration effect. In addition, compression mechanism is also discussed.

Schematic View of Disk MHD Accelerator

The schematic view of a disk-shaped MHD accelerator in the cylindrical coordinates is indicated in Fig.1.

As the air plasma with potassium which is utilized as a seed material, flows from the z-direction (flow-in), and the plasma eventually flows out in the r-direction. This is the principle of the “Outflow” system. The magnetic flux density B is created by the interaction of the upper coils and the lower coils. Without the input Hall Current, the j_θ normally flows in the negative θ-direction. This condition is the same as the MHD power generation. Then, the Lorenz force F_r (i.e. $\boldsymbol{j}\times\boldsymbol{B}$) operates in a reverse direction to the flow direction. As the external Hall current is applied to the negative radial direction as shown in Fig.1, and as induced Hall field is larger than Hall electromotive force, direction of the resultant Hall current flows in the positive θ-direction. In this case, F_r operates in the same direction of u_r. As a result, u_r is accelerated by this Lorenz force F_r. This is a principle of the Hall MHD acceleration for the disk-shaped channel. In general, there are three methods to connect electrode for MHD accelerators, namely, Faraday, Hall and Diagonal connection. In this study, the Hall connection is selected and presented.

The very interesting feature of a disk-shaped MHD accelerator is that plasma is able to flow in a reverse

direction. In this study, this flow direction is called "Inflow". The plasma flows in from the negative r-direction and flows out in z-direction. Therefore, the direction of applied Hall current is determined by the flow direction of the plasma flow.

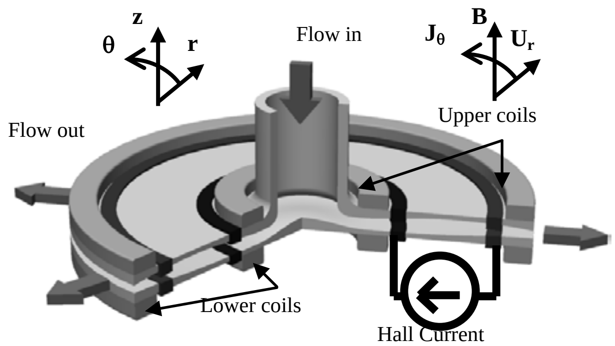

FIG. 1 SCHEMATIC VIEW OF OUTFLOW DISK MHD ACCELERATOR

Simulation Model

In this study, air-plasma is utilized as a working gas. The equilibrium plasma can be assumed where electron and gas temperatures are completely the same. And gas properties depend on both gas temperature and gas pressure. In particular, the influence of the temperature to the gas properties is more dependable. To cut down the calculation time, we utilized the numerical gas combustion program [11]. Acquired gas properties of the combustion of air plasma with potassium are applied to the Q1D unsteady simulation program.

Moreover, the MacCormack method is applied as the calculation scheme. Analysis region is only inside the MHD channel in this work, and as the initial condition, we gave isentropic flow to the MHD channel. Meanwhile, the back pressure is not considered.

Flow Field Equation

The flow field equations are described by as continuity equation, momentum equation, and energy equation. The equation of state is used additionally. The MHD effect is also added to the momentum equation as Lorentz force and the energy equation as Joule heating, and pushes work. These equations are expressed as follows,

$$\frac{\partial \rho A}{\partial t} + \nabla \cdot (\rho \vec{u} A) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \vec{u} A) + \nabla \cdot (\rho \vec{u} \vec{u} A) = (\vec{j} \times \vec{B} - \nabla P - P_L) A \tag{2}$$

$$\frac{\partial E_s A}{\partial t} + \nabla \cdot \{(E_s + p)\vec{u} A\} = \left(\frac{|\vec{j}|^2}{\sigma} + \vec{u} \cdot (\vec{j} \times \vec{B}) - Q_L \right) A \tag{3}$$

where P_L is the pressure loss caused by wall friction. ∇P means pressure gradient. The term Q_L indicates heat loss in the channel's wall. Heat loss Q_L is expressed by the following equations:

$$Q_L = S_t \rho |u| C_P (T_{aw} - T_w) \cdot \frac{D}{A} \tag{4}$$

$$S_t = \frac{1}{2} C_f P_r^{-\frac{2}{3}} \tag{5}$$

$$T_{aw} = \left(1 + \frac{\gamma - 1}{2} M^2 P_r^{\frac{1}{3}} \right) \cdot T_g \tag{6}$$

$$P_r = \frac{C_P \nu}{\kappa} \tag{7}$$

E_s is the total energy of the working gas, and is expressed as:

$$E_s = \rho \left(c_v T_g + \frac{1}{2} |\vec{u}|^2 \right) \tag{8}$$

The equation of state is expressed as:

$$P_g = \rho R T_g \tag{9}$$

Plasma Equation

The plasma equation consists of a momentum equation and the Maxwell equations with MHD approximation. An equation of electric charge and an energy equation for electrons are not required in the present calculation. The momentum equation (Generalized Ohm's equation) is expressed as follows:

$$\vec{j} + \frac{\beta}{B} \vec{j} \times \vec{B} = \sigma (\vec{E} + \vec{u} \times \vec{B}) \tag{10}$$

Here, electrical conductivity is evaluated by numerical combustion program [11], and depends on the gas temperature and pressure. In particular, it is strongly dependent upon gas temperature.

In this study, plasma is assumed to be under the following MHD approximations. 1) The magnetic Reynolds number is small. Therefore, the applied magnetic field is steady. 2) The Debye length is smaller than the distance of the plasma's property. This can be regarded as keeping the electrically

neutral of plasma. Moreover, since the plasma is not handling an extremely high frequency, the displacement of current can be negligible. As above-mentioned approximations, two simple equations are indicated as follows.

$$\nabla \times \vec{E} = 0 \tag{11}$$

$$\nabla \cdot \vec{j} = 0 \tag{12}$$

Simulation Parameter

The main purpose of the current study is to investigate the relationship of gas velocity and heat loss for various channel lengths. The disk-shaped MHD accelerator can use shorter MHD channel to increase the acceleration performance compare with linear MHD accelerators [9]-[10]. In addition, to use shorter MHD channel is to reduce the thermal loss because the surface area is proportional to the square of the radius.

We designed disk-shaped MHD channels with six different channel lengths between 0.045 m and 0.9 m, whose cross sectional area ratio of exit to inlet are the same. The radius of channel inlet is 0.05 m.

The analysis conditions and parameters are indicated in Table 1. The input power of 1 MW is applied to disk MHD accelerator. The duration of calculation was decided 2 ms because disk MHD accelerator becomes steady condition.

TABLE 1 SIMULATION PARAMETERS

Working gas	Air+K
Thermal input, MW	0.6
Flow direction	Outflow
Seed fraction, wt%	0.5
Applied magnetic field, T	2
Wall temperature, K	1000
Stagnation temperature, K	3530
Stagnation pressure, MPa	0.9144
Inlet Mach number	1.346
Swirl ratio	0
Input power, MW	1
Input Hall current, A	Variable
Channel length, m	0.045-0.9
Mesh space, mm	0.5

Results and Discussions

Relationship of Heat Loss and Channel Length

It is known that the cross sectional area of an outflow disk-shaped MHD channel can be expanded easily. However, the heat loss increases as the channel length elongates. Table 2 shows the calculation results for various channel length in reference [9]-[10] again. The definition of thermal loss T_{loss} is the ratio of amount of heat loss Q_L through the disk wall's to thermal and electrical inputs. And is expressed as follow;

$$T_{loss} = \frac{2 \cdot \int_{r_i}^{r_e} Q_L \, 2\pi r dr}{T_{input} + power_i} \times 100\% \tag{13}$$

The thermal loss for the channel length of 0.045 m is 9.36 %, while the value of 62.84 % is calculated for the channel length of 0.9 m. The results show that the thermal loss increases as the surface area of the channel increases. As far as the result of thermal loss for various channel length, the longer channel length resulted in the larger thermal loss. Hence we can expect that effective acceleration can be difficult for longer channel length, i.e. larger thermal loss.

TABLE 2 OBTAINED THERMAL LOSS AS VARYING THE CHANNEL LENGTH

Channel length, m	Channel volume, cm³	Channel surface area, cm²	Thermal loss, %	Acceleration efficiency, %
0.045	28	130	9.36	32.87
0.09	57	510	18.79	37.63
0.18	123	2040	31.99	29.70
0.27	201	4580	41.36	20.78
0.45	395	12720	52.66	10.19
0.9	1110	50890	62.84	1.46

Moreover as one of indicator for acceleration performance, acceleration efficiency η_e is introduced and expressed by as follow;

$$\eta_e = \frac{W_{fo} - W_{fi}}{power_i} \tag{14}$$

where inlet flow energy W_{fi} and outlet flow energy W_{fo} are shown as follow;

$$W_{fi} = \frac{1}{2} \rho u_r A u_r^{\,2} \text{ (at channel inlet)} \tag{15}$$

$$W_{fo} = \frac{1}{2}\rho u_r A u_r^{\ 2} \text{(at channel outlet)} \quad (16)$$

For the case of channel length of 0.9 m in Table 2, acceleration efficiency is 1.46 %, we found it is not accelerated effectively for this channel and it agrees with the inclination of thermal loss. However acceleration efficiency of the shortest channel of 0.045 m is up to 32.87 %. And it has no the best acceleration efficiency even if its thermal loss is the smallest. The cause will describe next section.

Gas Velocity and MHD Acceleration Effect

Figure 2 shows the gas velocity distributions for various channel length together with the isentropic distribution for the channel length of 0.9 m. Channel inlet radius of 0.05 m is also indicated in this figure. The maximum exit gas velocity of 3,390 m/s is calculated for the channel length of 0.09 m. in addition exit gas velocity for longer channel is decelerated corresponding to each thermal loss. Because of acceleration effect is obtained by thermal energy, MHD acceleration by Lorenz force and pressure gradient in reference [10]. Therefore the effect of first term is reduced or could not expect for longer channels. In particular, for the channel length of 0.9 m, the exit gas velocity is estimated to be 2,370 m/s, which almost corresponds to the gas velocity for isentropic case.

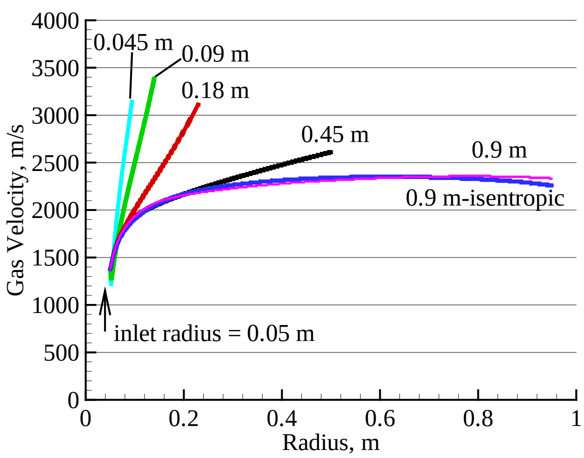

FIG. 2 RADIAL GAS VELOCITY FOR EACH MHD CHANNEL LENGTH

For the isentropic case, only the third acceleration effect affects gas velocity because no external input power and no loss for this case. For MHD acceleration case with input power, the third acceleration effect is also dominant for longer channel length of 0.9 m, because the input power and the Lorentz force are compensated due to larger thermal loss and larger friction loss. Therefore, acceleration effect of the first and the second terms are not significant. Thus, shorter MHD channel can achieve high acceleration performance mainly because of smaller thermal loss and smaller friction loss.

However the exit gas velocity is calculated as 3,240 m/s for the case of shortest channel length of 0.045 m, which is not as high as that for the channel length of 0.09 m case. The cause of the velocity drop can be ascribed to the decrease of gas velocity at the channel inlet, where plasma is compressed rather than accelerated.

Figure 3 shows distributions of gas velocity, static gas pressure and Faraday current density for the shortest channel length of 0.045 m. We can see clearly strong decrease in velocity and corresponding increase in static pressure at the inlet of the channel. This is due to large Faraday current density.

Faraday current density depends on the applied Hall current density and electromotive force in θ-direction. And then Hall current density became large for shorter MHD channel due to constant input power of 1 MW. Therefore, Faraday current density for shorter channel is also extremely large particularly for the channel length of 0.045 m in the present study. Local excessive Faraday current density at the inlet of channel significantly increases local gas temperature and static pressure increase dramatically by Joule heating.

Compression Phenomena and Its Mechanism

Sakamoto et al called this phenomenon is the MHD compression [8]. We can understand that too short channel length provides degradation of acceleration performance owing to the compression due to excessive Faraday current density, although the shorter channel length generally provides the higher acceleration performance due to smaller thermal and friction losses.

Sakamoto have solved MHD compression has two types to compress the gas pressure, i.e. they are pressure gradient and density gradient. According to the profiles of gas pressure and fluid density, the feature of pressure gradient is to increase gas pressure along the MHD channel only. On the other hand, density gradient is not only gas pressure but also fluid density increased at that same time.

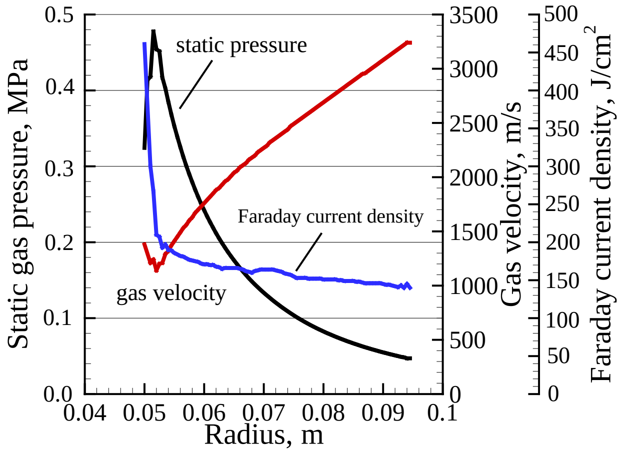

FIG. 3 DISTRIBUTION OF GAS STATIC PRESSURE, FARADAY CURRENT DENSITY AND GAS VELOCITY FOR 0.045 M

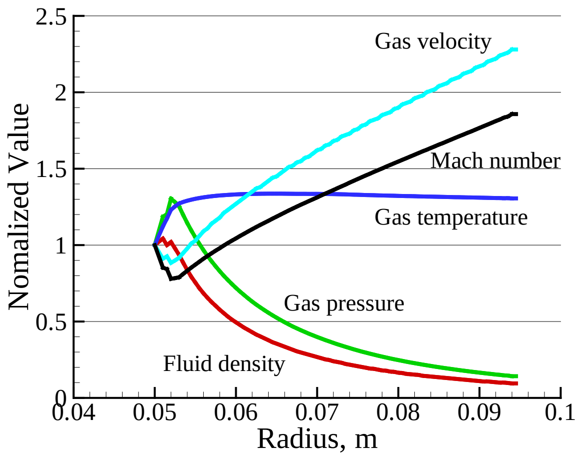

FIG. 4 NORMALIZED VALUE FOR EACH PARAMETER ALONG MHD CHANNEL OF 0.045 M

As one of sample, normalized value for some parameters are shown in Fig.4. The profiles of gas pressure and fluid density are increased at the close to the MHD channel inlet. In this case, this phenomenon is categorized to density gradient for MHD compression. His indication shows MHD compression is also generated at MHD channel inlet for disk MHD accelerator of Hall type connection, furthermore linear MHD accelerator, and disk MHD accelerator for faraday or diagonal connection have the possibility to generate the MHD compression midstream and downstream of the MHD channel in some conditions.

To avoid MHD compression at MHD channel inlet, it is available to set the MHD channel which has large cross sectional area [10]. And then Faraday current density j_θ is decreased corresponding to the cross sectional area, Lorenz force operates reversely for too low Faraday current density. Thus we need to consider MHD compression in the MHD channel to obtain high acceleration performance and design indication. Sakamoto's work is focused on the case of Faraday connection, therefore MHD compression for Hall type connection is needed to discuss in near the future.

Conclusions

This paper presented the fundamental results of acceleration performance of the disk-shaped MHD accelerator using by Q1D numerical simulation. Conclusions are as follows:

For the longest channel length of 0.9 m, thermal loss was estimated over 60 % and the exit gas velocity was almost the same as that for isentropic case. Acceleration could not observe owing to large heat loss and large friction loss. And then the acceleration efficiency was only about 1 %.

For shorter channel lengths of 0.045 m and 0.09 m, thermal loss can be reduced below 20 %. The shorter channel length could achieve higher acceleration performance owing to the smaller heat loss and the friction loss. Therefore both acceleration efficiencies are 32.9 % and 37.6 % respectively. However, the shortest channel length of 0.045 m, accelerator performance was decreased by the MHD compression owing to excessive Faraday current density.

Generation mechanism of MHD compression has pressure gradient and density gradient, and both gas pressure and fluid density were increased at that same time in the region of MHD compression. Hence the density gradient operated the compression in this work.

It is necessary to design as long as shorter channel length to prevent MHD compression at the inlet of channel for disk MHD accelerator. For example, to increase cross sectional area, to adjust the input power and to set multiple electrode pairs to control Faraday current density. Even if these solutions are installed, we have to consider "stable acceleration" along the MHD channel to the design of disk MHD accelerator.

Nomenclature

u	:gas velocity, m/s
E	:electric field, V/m
j	:current density, A/m^2
σ	:electrical conductivity, S/m
β	:Hall parameter
ρ	:fluid density, kg/m^3

B	:magnetic flux density, T
T	:temperature, K
P	:pressure, Pa
A	:cross sectional area, m^2
D	:channel diameter, m
Cv	:specific heat in constant volume, J/kg/K
C_P	:specific heat in constant pressure, J/kg/K
P_L	:pressure loss, Pa
Q_L	:heat loss, W/m
C_f	:local cross-section friction coefficient
M	:Mach number
S_t	:Stanton number
P_r	:Prandtl number
v	:viscosity, Pa s
κ	:thermal conductivity, J/s/m/K
T_{loss}	:thermal loss, W
$power_i$	:input power, W
T_{input}	:thermal input, W
r_i	:channel inlet radius, m
r_e	:channel exit radius, m
W_{fi}	:inlet flow energy, MW
W_{fo}	:outlet flow energy, MW

Subscripts

0	:stagnation
g	:gas
aw	:adiabatic wall
w	:wall
r	:r-direction
θ	:θ-direction

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